

Methods of Proof I

Key Topics

- * What is a Proof?
 - * Formal vs. Informal Proofs
 - * Proofs with Identity
 - * Carryover to Other Predicates
 - * Formal Proofs & Fitch
 - * Proofs of Nonconsequence
-

- Is this a valid argument?

The President of the United States must be at least 35 years old.
Bill is at least 35 years old.
Therefore, Bill is President of the United States.

- Proof: a step-by-step demonstration that a conclusion follows from some premises.

Premises

- a. If my glasses are on the kitchen table, then I saw them at breakfast.
- b. I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
- c. If I was reading the newspaper in the living room, then my glasses are on the coffee table.
- d. I did not see my glasses at breakfast.
- e. If I was reading my book in bed, then my glasses are on my bed table.
- f. If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

Conclusion: My glasses are on the coffee table.

Some intermediate conclusions - how did we arrive at these?

- g. The glasses are not on the kitchen table
- h. I did not read the newspaper in the kitchen
- i. I read the newspaper in the living room

- Formal Proof vs. Informal Proofs

Remember: All proofs must be *rigorous*, i.e., each step in a proof must provide definitive evidence that the intermediate conclusion follows from things already established.

Formal Proof: Every step in the proof is provided (i.e., no steps are left out), a fixed set of rules are used as explanations of intermediate conclusions; usually presented in a highly stylized, formal way.

Informal Proof: Usually stated in English, in paragraph form; less formal and the more obvious steps are left out. (Which steps can be left out?)

(We will be much more exact about these categories later...)

- Proofs with Identity

= Elim: If $b = c$ then whatever holds for b , holds for c .

= Intro: $b = b$ is always true (reflexivity)

Symmetry of Identity: If $b = c$, then $c = b$.

Transitivity of Identity: If $a = b$ and $b = c$, then $a = c$.

- Carryover to Other Predicates

In the blocks FOL:

Larger is transitive; SameRow is reflexive and symmetric

Inverses: b is larger than c , so c is smaller than b ; larger and smaller are "inverses", so are right/left. They refer to the same relation but in opposite directions.

- Proofs of Nonconsequence

Proving that a conclusion does *not* follow from the premises.

An invalid argument is one where there is some circumstance that make the premises true but the conclusion false - we just have to find the circumstance. This is called a counterexample.